Name: ______________________________

# GCSE (1 – 9)

# Circle Theorems

## Instructions

- Use **black** ink or ball-point pen.
- Answer all questions.
- Answer the questions in the spaces provided

*– there may be more space than you need.*
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working out.**

## Information

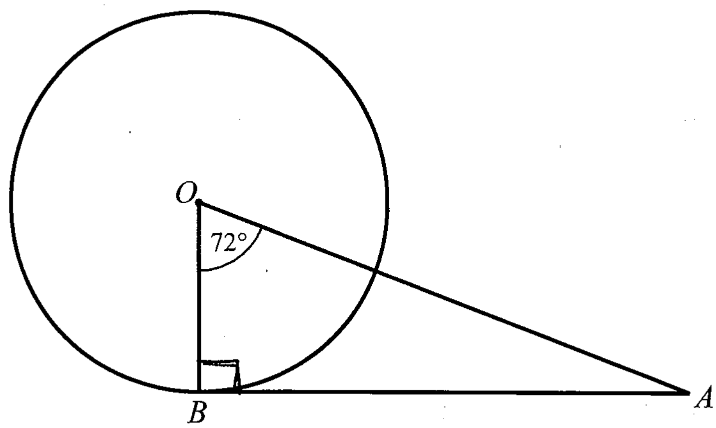
- The marks for each question are shown in brackets

*– use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end

**1**




$B$ is a point on the circumference of a circle, centre $O$.
$AB$ is a tangent to the circle.

Angle $BOA = 72°$

Work out the size of angle $BAO$.
You must show all your working.

OBA = 90° Tangent meets radius at 90°

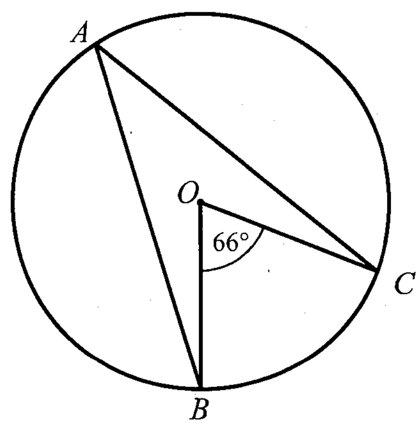
BAO = 180 − 90 − 72 Angles in a triangle add to 180°

= 18°

18 °

**(Total for Question 1 is 2 marks)**

**2**




$A$, $B$, $C$ and $D$ are points on the circumference of a circle.

Angle $BOC = 66°$

(i) Find the size of angle $BAC$.

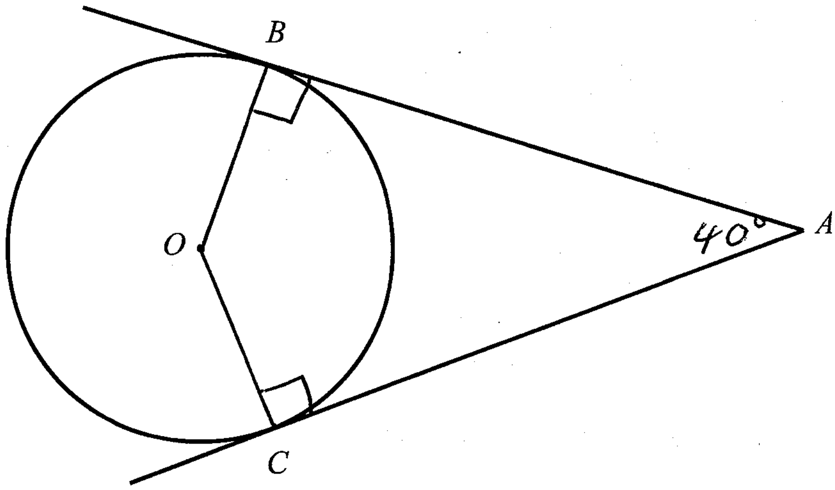
33 °

(ii) Give a reason for your answer.

The angle at the circumference is half the angle at the centre

**(Total for Question 2 is 2 marks)**

**3**




$B$ and $C$ are points on a circle, centre $O$.
$AB$ and $AC$ are tangents to the circle.

Angle $BAC = 40°$

Work out the size of angle $BOC$.
You must show all your working.

OBA and OCA = 90°
Tangent meets radius at 90°

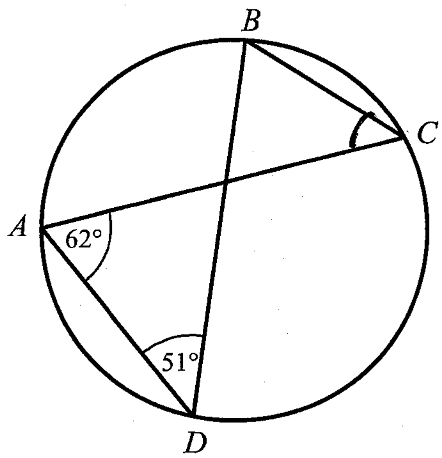
360 − 90 − 90 − 40 = 140°
Angles in a quadrilateral add to 360°

140 °

**(Total for Question 3 is 3 marks)**

---

**4**




$A$, $B$, $C$ and $D$ are points on the circumference of a circle.

Angle $CAD = 62°$
Angle $ADB = 51°$

(i) Find the size of angle $ACB$.

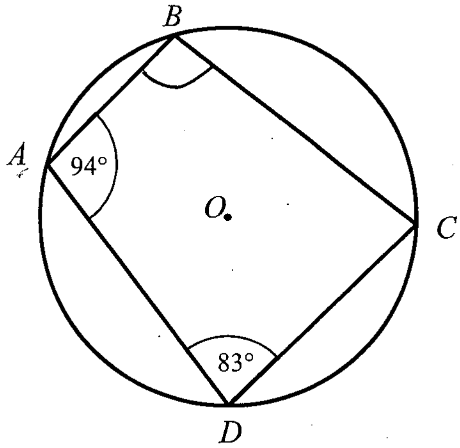
51 °

(ii) Give a reason for your answer.

Angles in the same segment are equal.

**(Total for Question 4 is 2 marks)**

**5**




*A, B, C* and *D* are points on the circumference of a circle.

Angle $BAD = 94°$
Angle $ADC = 83°$

180 − 83

(i) Find the size of angle *ABC*.

97 °

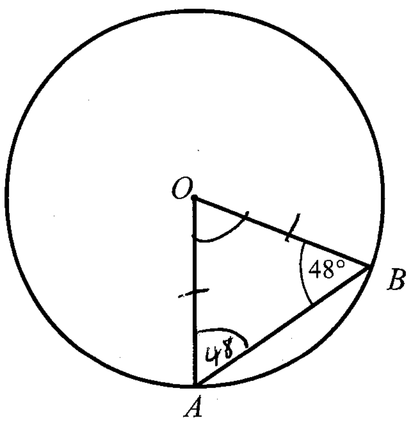
(ii) Give a reason for your answer.

opposite angles in a cyclic quadrilateral add to 180°

**(Total for Question 5 is 2 marks)**

---

**6**




OAB = 48°
Angles at base of isosceles are equal

*A* and *B* are points on the circumference of a circle, centre *O*.

Angle $ABO = 48°$

(i) Find the size of angle *AOB*.

180 − 48 − 48

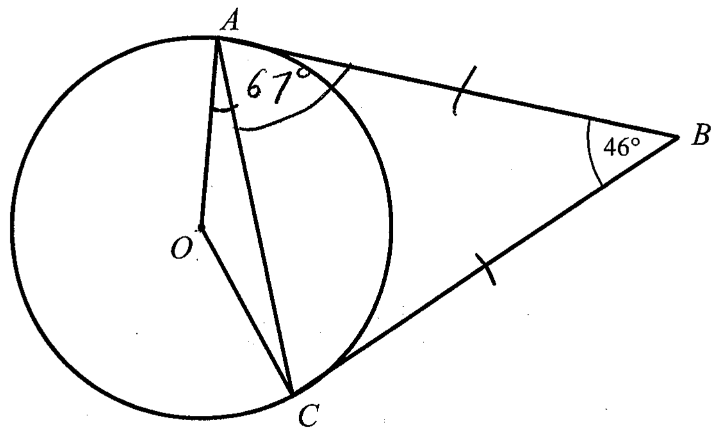
84 °

(ii) Give a reason for your answer.

Angles at the base of an isosceles triangle are equal + angles in a triangle add to 180°

**(Total for Question 6 is 2 marks)**

**7**




*A* and *C* are points on the circumference of a circle, centre *O*.
*AB* and *BC* are tangents to the circle.

Angle *ABC* = 46°

Find the size of angle *OAC*.
Give reasons for each stage of your working.

ABC is an isosceles triangle, 2 tangents from the same point are equal

$$CAB = \frac{180 - 46}{2} = \frac{134}{2} = 67°$$

Angles at the base of an isosceles triangle are equal

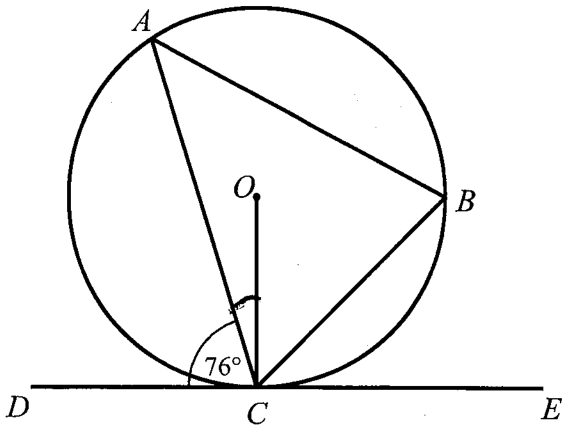
OAB = 90° Tangent meets radius at 90°

OAC = 90 − 67 = 23°

23 °

**(Total for Question 7 is 4 marks)**

**8**




*A* and *B* are points on the circumference of a circle, centre *O*.
*DCE* is a tangent to the circle.

Angle *ACD* = 76°

(a) Find the size of angle *ACO*.
You must ~~show~~ all your working.
give reasons for

OCD = 90° Tangent meets radius at 90°

90 − 76 = 14°

14 °
**(2)**

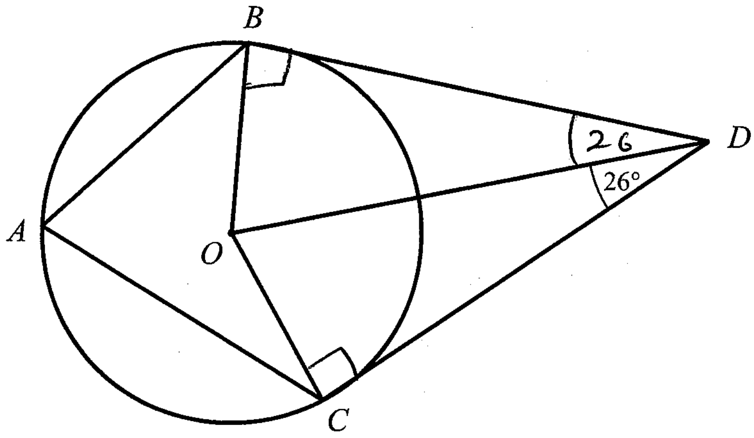
(b) Find the size of angle *ABC*.
You must ~~show~~ all your working.
give reasons for

ABC = 76° Alternate segment theorem

76 °
**(2)**

**(Total for Question 8 is 4 marks)**

**9**




*A, B* and *C* are points on the circumference of a circle, centre *O*.
*BD* and *CD* are tangents to the circle.

Angle *ODC* = 26°

Find the size of angle *BAC*.
Give reasons for each stage of your working.

Triangle BOD = Triangle DOC  all sides are equal

OBD and OCD = 90°  Tangent meets radius at 90°

BOD and COD = 180 − 90 − 26
= 64°

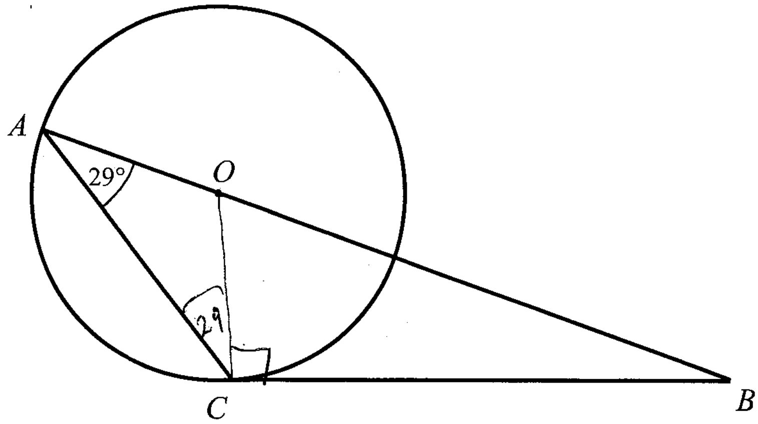
BOC = 2 × 64 = 128°

BAC = $\frac{128}{2}$ = 64°  Angle at circumference is half the angle at the centre

64 °

**(Total for Question 9 is 4 marks)**

**10**




*A* and *C* are points on the circumference of a circle, centre *O*.
*BC* is a tangent to the circle.

Angle *CAB* = 29°

Find the size of angle *ABC*.
You must show all your working.

OCA = 29° Angles at the base of an isosceles triangle are equal

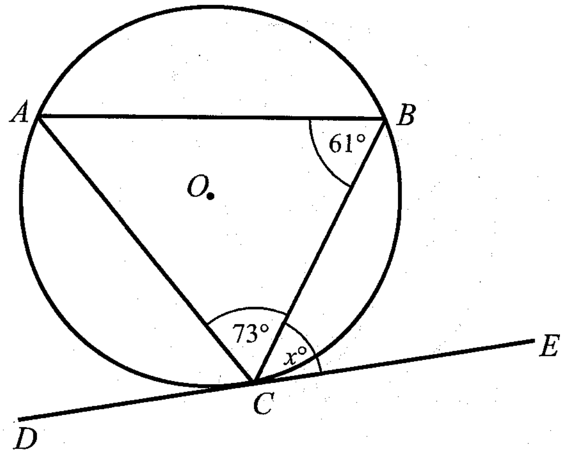
OCB = 90° Tangent meets radius at 90°

ABC = 180 − 90 − 29 − 29
= 32° Angles in a triangle add to 180°

32 °

**(Total for Question 10 is 4 marks)**

**11**




*A*, *B* and *C* are points on the circumference of a circle, centre *O*.
*DCE* is a tangent to the circle.

Angle $ABC = 61°$
Angle $ACB = 73°$
Angle $BCE = x°$

Find the value of $x$.
You must show all your working.

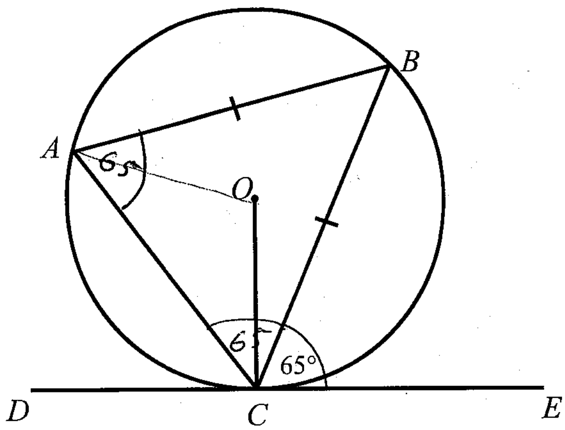
BAC = 180 − 73 − 61
= 46° Angles in a triangle add to 180°

x = 46° Alternate segment theorem

46 °

**(Total for Question 11 is 3 marks)**

**12**




*A*, *B* and *C* are points on the circumference of a circle, centre *O*.
*DCE* is a tangent to the circle.

$AB = BC$
Angle $BCE = 65°$

Find the size of angle *AOC*.
You must show all your working.

BAC = 65° Alternate segment theorem

ACB = 65° Angles at the base of an isosceles triangle are equal

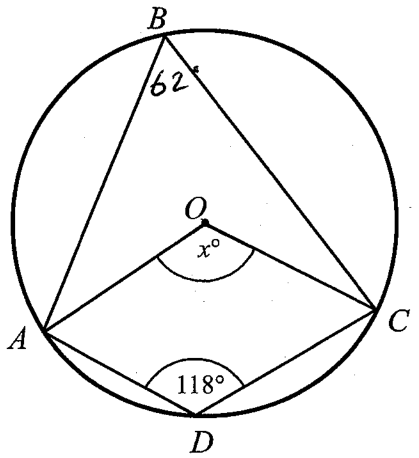
ABC = 180 − 65 − 65
= 50° Angles in a triangle add to 180°

AOC = 2 × 50
= 100° Angle at centre is twice angle at circumference

100 °

**(Total for Question 12 is 4 marks)**

**13**




*A, B, C* and *D* are points on the circumference of a circle, centre *O*.

Angle $ADC = 118°$
Angle $AOC = x°$

Work out the value of *x*.
You must show all your working.

ABC = 180 − 118
= 62°

opposite angles in a cyclic quadrilateral add to 180°

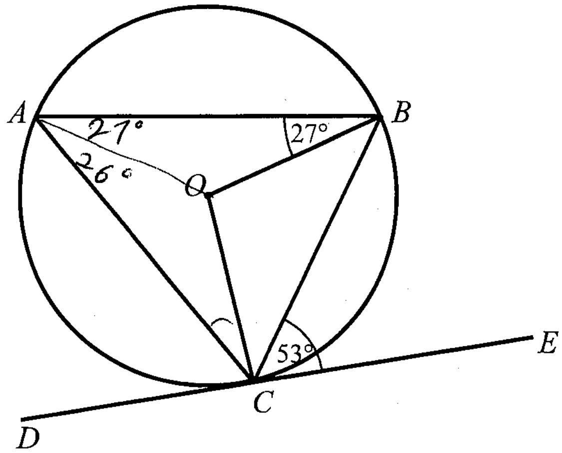
AOC = 2 × 62
= 124°

Angle at centre is twice angle at circumference

.......124.......°

**(Total for Question 13 is 3 marks)**

**14**




*A*, *B* and *C* are points on the circumference of a circle, centre *O*.
*DCE* is a tangent to the circle.

Angle $ABO = 27°$
Angle $BCE = 53°$

Find the size of angle *ACO*.
~~You must show all your working~~.
Give reasons for each stage of your working

BAO = 27° Angles at base of isosceles triangle are equal

BAC = 53° Alternate segment theorem

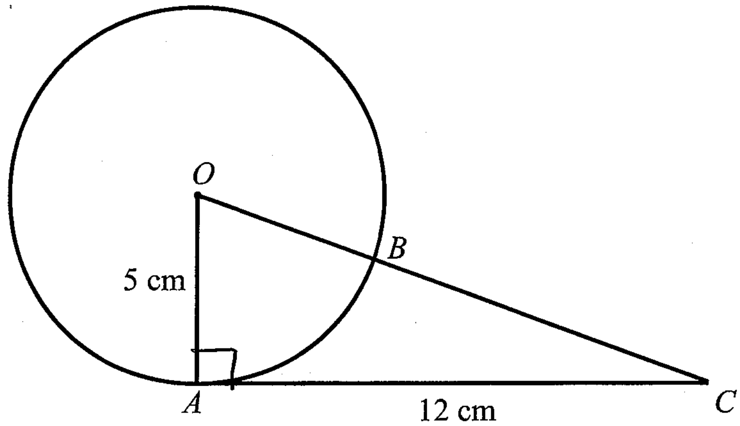
OAC = 53 − 27 = 26°

ACO = 26° Angles at base of isosceles triangle are equal

26 °

**(Total for Question 14 is 4 marks)**

**15**




*A* and *B* is a point on the circumference of a circle, centre *O*.
*AC* is a tangent to the circle.
*OBC* is a straight line.

*OA* = 5 cm
*AC* = 12 cm

Find the length of *BC*.
You must show all your working.

OAC = 90° Tangent meets radius at 90°

$5^2 + 12^2 = OC^2$

$169 = OC^2$

$OC = \sqrt{169}$

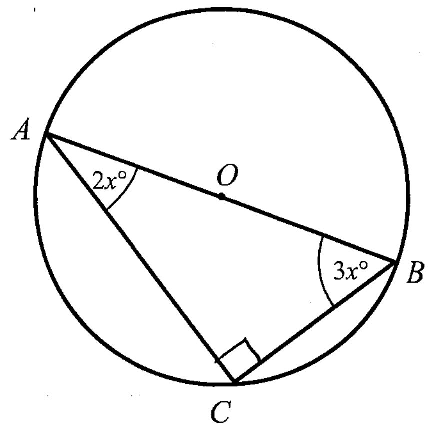
$= 13$ cm

BC = 13 − 5 = 8 cm (radius = 5cm)

8 cm

**(Total for Question 15 is 4 marks)**

**16**




$A$, $B$ and $C$ are points on the circumference of a circle, centre $O$.

Angle $CAB = 2x°$
Angle $ABC = 3x°$

Find the value of $x$.
You must show all your working.

ACB = 90° Angle in a semi circle is 90°

$2x + 3x + 90 = 180$

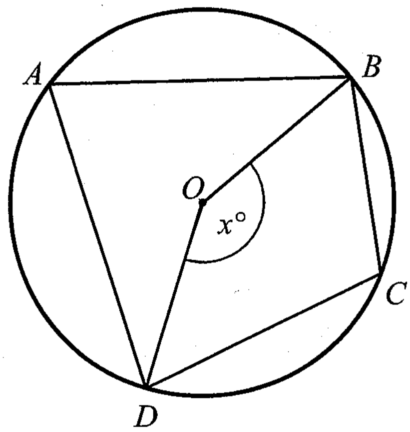
$5x = 90$

$x = 18°$

$x =$ 18

**(Total for Question 16 is 3 marks)**

**17**




*A, B, C* and *D* are points on the circumference of a circle, centre *O*.

Angle $BOD = x°$

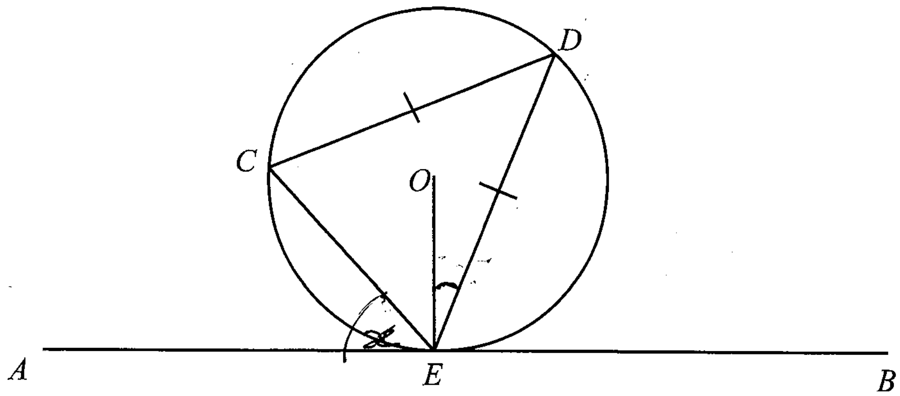
Find the size of angle *BCD*, in terms of *x*.
Give reasons for each stage of your working.

$BAD = \frac{1}{2}x$ Angle at circumference is half angle at centre

$BCD = 180 - \frac{1}{2}x$ opposite angles in a cyclic quadrilateral add to 180°

**(Total for Question 17 is 3 marks)**

**18**




$C$, $D$ and $E$ are points on a circle, centre $O$.
$AEB$ is a tangent to the circle at $E$.

$CD = DE$
Angle $AEC = x°$

Find the size of angle $OED$, in terms of $x$.
Give reasons for each stage of your working.

CDE = x  Alternate segment theorem

CED = $\frac{180 - x}{2}$  Angles at the base of an isosceles triangle are equal

OEC = 90 − x  OEA = 90° Tangent meets radius at 90°

OED = $\frac{180 - x}{2} - (90 - x)$

$= 90 - \frac{1}{2}x - 90 + x$

$= \frac{1}{2}x$

**(Total for Question 18 is 5 marks)**